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Does Magnetic Charge Imply a Massive Photon?

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In Abelian monopole theories the magnetic coupling is required to be enormous. Using the electric–magnetic duality of electromagnetism, it is argued that the existence of such a large, nonperturbative magnetic coupling should lead to a phase transition where magnetic charge is permanently confined and the photon becomes massive. The apparent masslessness of the photon could then be used as an argument against the existence of such a large, nonperturbative magnetic charge. Finally it is shown that even in the presence of this conjectured dynamical mass generation the Cabbibo–Ferrari (1962) formulation of magnetic charge gives a consistent theory.

1. STRONG COUPLING PHASE TRANSITION

Normally the gauge bosons of a theory are said to be massless due to the requirement of gauge invariance. If the Lagrangian of a theory has a mass term for the gauge bosons (i.e., a term like $\frac{1}{2}m^2 A_\mu A^\mu$), then the Lagrangian is no longer invariant under the gauge transformation of the gauge field [i.e., $A_\mu \rightarrow A_\mu - (1/e)\partial_\mu \Lambda(x)$, where $\Lambda(x)$ is an arbitrary function]. One escape from this prohibition is the Higgs mechanism (Higgs, 1964a,b, 1966), which allows the gauge boson to have a mass while still remaining consistent with gauge invariance. This is accomplished by coupling the gauge boson to a scalar field which develops a vacuum expectation value. A less often stated restriction is that the coupling of the gauge boson to particles of the theory needs to be small enough (Huang, 1982) so that the gauge boson does not become massive through some nonperturbative mechanism (e.g., the technicolor models for mass generation in the standard model). It is difficult to give a definite value for how small the coupling constant should be in order to ensure the masslessness of the gauge boson, but requiring that it be small enough so that perturbation theory is valid seems a good rule of thumb.

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Wilson (1974) argued that in a $U(1)$ gauge theory there should be some critical coupling e_c below which the $U(1)$ gauge boson is massless and the charges are free, and above which the gauge boson becomes massive and charges are confined. Wilson's conjecture does not determine whether this phase transition from massless gauge boson to massive gauge boson is a first- or second-order transition, nor does it give the value of the critical coupling at which this transition should occur. This conjectured mechanism, which dynamically generates a mass for the $U(1)$ gauge boson, is similar to an effect which was found to occur in QED in 1 + 1 dimensions. Schwinger (1962a,b) rigorously showed that in 1 + 1 dimensions the photon would acquire a mass proportional to e^2, the square of the coupling. Thus in (1 + 1)-dimensional QED $e_c = 0$, and the photon always becomes massive. Schwinger also conjectured that the same effect could occur in 3 + 1 QED for some unspecified, large coupling. Guth (1980) showed that a $U(1)$ gauge theory will indeed undergo a phase transition as conjectured by Wilson and Schwinger, but no theoretical value for the critical coupling constant was given. Thus for (3 + 1)-dimensional QED it may be an "accident" of the gauge coupling e being small that results in the physical photon being massless within very stringent limits [the upper bound on the photon mass is 3.0×10^{-27} eV = 5.3×10^{-63} kg $> m_\gamma$ (Particle Data Group, 1994)]. The amazing success of perturbation theory for the electromagnetic interactions of the electron also indicates that the physical electromagnetic coupling is below this unknown critical value. QCD, in contrast, is thought to exist in the confining phase with a fine structure constant $\alpha_s = g_s^2/4\pi$ on the $\mathcal{O}(1)$.

In Dirac's theory of magnetic charge one allows the vector potential $\mathbf{A}$ to develop a singularity that runs from the location of the magnetic charge to spatial infinity, so that $\nabla \cdot \mathbf{B} = \rho_m$ is consistent with the $\mathbf{B} = \nabla \times \mathbf{A}$ (Dirac, 1931, 1948). Dirac also showed that in order for the wavefunction of an electrically charged particle in the presence of this string singularity to be single-valued, the following quantization condition had to hold:

$$\frac{eg}{4\pi} = \frac{n}{2} \tag{1}$$

where n is an integer, g is the magnitude of the magnetic charge, and e is the magnitude of the electric charge (which we will take to be the charge of the electron). There are other ways of formulating a theory of magnetic charge without having to take recourse to a singular vector potential [the fiber bundle approach of Wu and Yang (1975) or the two-potential approach of Cabbibo and Ferrari (1962)]. In all these various theories, however, one eventually ends up with a similar quantization condition. The best model-independent argument for this is due to Saha (Saha, 1936, 1949; Wilson, 1949). If one considers a particle with electric charge e in the presence of a particle with magnetic charge g, then due to the $\mathbf{E} \times \mathbf{B}$ term in the energy-momentum tensor this system carries a field angular momentum of magnitude $eg/4\pi$. Since angular momentum is quantized in integer multiples of $\hbar/2$, we again arrive at condition (1), where we have set $\hbar = 1$.

If e in equation (1) is taken as the physical charge of the electron, it is found that the magnitude of the magnetic charge is enormous. The strength of the electric coupling strength between two electric charges is $e^2/4\pi \approx 1/137$, while the strength of the minimum magnetic coupling [i.e., $n = 1$ in equation (1)] between two monopoles is $g^2/4\pi \approx 137/4$. The interaction strength between two monopoles is roughly 5×10^3 times stronger than that between two electric charges. The size of the magnetic coupling puts it well out of the range of perturbation theory, and opens up the logical possibility that unusual nonperturbative effects could occur in the presence of such a nonperturbative magnetic charge. In Wilson and Guth's argument for a phase transition in a $U(1)$ gauge theory with a large coupling, the $U(1)$ gauge charge is usually thought of as electric charge. If the $U(1)$ gauge charge is taken to be electric charge, then there is a definite difference, in the standard formulation of the theory, in the way the gauge boson couples to electric charge as compared to how it couples to magnetic charge. The electric charge is minimally coupled to the vector potential A_μ, while the magnetic charge has no simple coupling A_μ. Physically, however, the gauge boson should couple to both charges in a symmetric way, especially when one looks at the theory in terms of how these charges interact with the $\mathbf{E}$ and $\mathbf{B}$ fields. This physically intuitive idea takes the mathematical form of a dual symmetry between electric and magnetic quantities. For the $\mathbf{E}$ and $\mathbf{B}$ fields, the dual symmetry is (Jackson, 1975)

$$\begin{aligned} \mathbf{E} &\rightarrow \mathbf{E} \cos\theta + \mathbf{B} \sin\theta \\ \mathbf{B} &\rightarrow -\mathbf{E} \sin\theta + \mathbf{B} \cos\theta \end{aligned} \tag{2}$$

For the electric and magnetic charge and current densities ($J_e^\mu = (\rho_e, \mathbf{J}_e)$ and $J_m^\mu = (\rho_m, \mathbf{J}_m)$) the dual symmetry is

$$\begin{aligned} J_e^\mu &\rightarrow J_e^\mu \cos\theta + J_m^\mu \sin\theta \\ J_m^\mu &\rightarrow -J_e^\mu \sin\theta + J_m^\mu \cos\theta \end{aligned} \tag{3}$$

Maxwell's equations with magnetic sources are invariant under the combined action of (2) and (3). This dual symmetry between electric and magnetic charges and currents shows that it is a matter of convention as to what is called electric charge and what is called magnetic charge. In fact Baker *et al.* (1995) have shown that electromagnetism can be reformulated with magnetic charge as the gauge charge, while electric charge is attached to Dirac-type strings. In this dual reformulation of electromagnetism it is the magnetic charge which is minimally coupled to the $U(1)$ gauge boson. Thus Wilson and Guth's conjecture about a phase transition for strongly coupled $U(1)$ theory to a confining phase with a massive gauge boson should be applicable to this dual reformulation of electromagnetism where the magnetic charge is directly coupled to the photon. The only unresolved question is whether or not the large value of the magnetic coupling is greater than the unknown critical coupling necessary to cause this phase transition.

Combining this dual symmetry with the conjecture of a strong-coupling phase transition to a confining phase with a massive gauge boson, it can be argued that a large, nonperturbative magnetic charge would make the photon massive. The dual symmetry is important since it indicates that it should not make a difference whether the large, nonperturbative charge is electric or magnetic. Since the photon is apparently massless to some stringent upper limit, this implies that Abelian magnetic charge is absent from the physical world. As we shall see, the Cabbibo–Ferrari formulation of magnetic charge could still give a consistent theory even in the presence of this dynamical mass generation. Even though there is no theoretical prediction as to the critical value of the coupling at which this phase transition should occur, the value at which QCD apparently undergoes this phase transition, while not exactly determined, is certainly thought to be much less than 137/4. A rough estimate of the critical coupling can be made by considering the free energy of Wilson loops for a $U(1)$ gauge theory (Kogut, 1983). This gives a critical coupling of approximately $g_c^2 \approx 1.57$, which is smaller than the required strength of the magnetic charge from equation (1). This rough estimate of the critical coupling agrees with numerical work on compact lattice $U(1)$ gauge theory, which points to a critical coupling of the order unity (DeGrand and Toussaint, 1980; Lautrup and Nauenberg, 1980). Assuming that as the limit of the lattice spacing is taken to zero the lattice theory goes over smoothly into the continuum theory, one again finds an indication that the required value of the magnetic coupling is in the confinement regime where the gauge boson is massive.

The coexistence of confinement and massive gauge bosons (the Higgs mechanism) may seem strange, since massive gauge bosons usually imply a short-range effect. However, Fradkin and Shenker (1979) investigated the phase diagram for a $U(1)$ theory with arbitrary coupling and found that the Higgs phase and the confinement phase do indeed coexist so long as the $U(1)$ coupling is greater than its critical value.

2. THE TECHNICOLOR ANALOGY

In this section we will give an argument, based on an analogy to the technicolor idea, that also points to the possibility that in the presence of magnetic charge the photon would develop a dynamical mass. The basic idea behind technicolor theories is to introduce a new set of fermions (i.e., techni-fermions) which couple to a new, strong, non-Abelian gauge force called technicolor. The techni-fermions form a condensate, $\langle \overline{F}F \rangle \neq 0$, which gives the theory a vacuum expectation value. The elementary Higgs scalar is replaced by the composite scalar $\overline{F}F$, which must have the correct quantum numbers in order to mix with the gauge boson that is to become massive. Even without a technicolor interaction it is thought that QCD by itself gives the same dynamical mass generation via the mixing of the $SU(2)$ gauge bosons with light quark–antiquark condensates. The problem with this is that the scale of the QCD interaction gives a mass to the gauge bosons which is several orders of magnitude too small. Thus one must introduce the QCD-like technicolor interactions which are postulated to have the right scale in order to give the $SU(2)$ gauge bosons masses on the order of 80–90 GeV. In the present case instead of the composite scalar being composed of techni-fermions it is composed of a monopole–antimonopole pair. Denoting the monopole–antimonopole condensate by Π_m, we can, in analogy with technicolor, introduce an effective coupling between the photon and this composite scalar particle

$$\mathcal{L}_{\gamma-m} = \frac{f_m}{2} (gA^{\mu})(\partial_{\mu}\Pi_m) \tag{4}$$

where f_m is a constant, which is the equivalent of the pion decay constant of QCD. This interaction term in the Lagrangian mixes the photon with the composite Π_m with a Feynman rule vertex of $-(igf_m/2)q_{\mu}$, where q_{μ} is the momentum of the photon. Taking an infinite sum of Π_m's mixing in with the photon changes the photon's propagator from

$$D^{\gamma}_{\mu\nu} = \frac{-i(g^{\mu\nu} - q_{\mu}q_{\mu}/q^2)}{q^2} \tag{5}$$

to

$$D^{\gamma}_{\mu\nu} = \frac{-i(g^{\mu\nu} - q_{\mu}q_{\mu}/q^2)}{q^2 - g^2 f_m^2/4} \tag{6}$$

The pole in the second propagator indicates that the photon now has a mass of $m_{\gamma} = gf_m/2$. This mass is arbitrary since the "magnetic" pion decay constant f_m is unspecified. Both the argument based on Wilson and Guth's idea of a phase transition for a strongly coupled theory and this more heuristic technicolor-inspired argument point to the photon developing a mass in the presence of a large magnetic charge. Both arguments have a degree of ambiguity. In the first case the critical value at which the phase transition occurs is not determined theoretically; in the second case the mass given to the photon is arbitrary since it depends on the unknown "magnetic" pion decay constant f_m. In either case one could still make the argument that the mass given to the photon by the nonperturbative magnetic charge is smaller than the experimental upper limit on the photon mass. Given the stringent upper bound on the photon mass, this argument is unnatural. The more likely statement is that the apparent masslessness of the photon implies the absence of magnetic charge.

3. DISCUSSION AND CONCLUSIONS

Using two different approaches, we have argued that the required large, nonperturbative value of magnetic charge is inconsistent with the apparent masslessness of the photon. Or put in reverse: the apparent masslessness of the photon implies the absence of magnetic charge with the large, nonperturbative coupling which is required in Abelian monopole theories. This statement is too broad. The Cabbibo–Ferrari formulation of magnetic charge could still remain consistent with this dynamical mass generation for the photon *if* one interprets the second potential as a second gauge boson. In the Cabbibo–Ferrari approach a second pseudo four-vector potential $C_\mu = (\phi_m, \mathbf{C})$ is introduced in addition to the usual four-vector potential $A_\mu = (\phi_e, \mathbf{A})$. Then in terms of these two potentials the normal definitions of the **E** and **B** fields get expanded to

$$E_i = F^{0i} - \mathcal{G}^{0i}, \qquad B_i = G^{0i} + \mathcal{F}^{0i} \tag{7}$$

where the field strength tensors are

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \qquad G_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu \tag{8}$$

and their duals are

$$\mathcal{F}_{\mu\nu} = \frac{1}{2}\,\epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma}, \qquad \mathcal{G}_{\mu\nu} = \frac{1}{2}\,\epsilon_{\mu\nu\rho\sigma} G^{\rho\sigma} \tag{9}$$

It is now possible to cast the dual relationship between the **E** and **B** fields of (2) in terms of the four-vector potentials,

$$\begin{aligned} A_\mu &\to A_\mu \cos\theta + C_\mu \sin\theta \\ C_\mu &\to -A_\mu \sin\theta + C_\mu \cos\theta \end{aligned} \tag{10}$$

making the theory dual at the level of the four-potentials. Even though there are two potentials in this approach, one normally imposes conditions on these two potentials so that in the end there are only enough degrees of freedom left to account for one photon (Zwanziger, 1971). In the Cabbibo–Ferrari theory one also ends up with an enormous, nonperturbative value for the magnetic coupling due to Saha's angular momentum quantization argument. Thus, in the one-photon version of the Cabbibo–Ferrari formulation, the apparent observed masslessness of the photon again implies the absence of magnetic charge. If, however, the pseudo four-vector potential is taken to be a second, parity-odd photon, then a consistent theory can be given even in the presence of a large, nonperturbative magnetic coupling. One can arrange for the dynamical symmetry breaking to give a mass to the pseudo photon C_μ while the second photon A_μ remains massless. This is in direct analogy with what happens in the $SU_L(2) \times U(1)$ standard model, where the Z boson becomes massive while the photon remains massless. This happens whether the symmetry breaking is spontaneous or dynamical. Thus, taking C_μ as a real gauge boson not only allows one to have a nonperturbative magnetic coupling, but also naturally explains the absence of this second pseudo photon from the particle spectrum that has so far been probed. Most work on the Cabbibo–Ferrari theory of magnetic charge takes the view of Zwanziger (1971) that there is only one photon. However, there is some work that does regard the potential C_μ as being a second, physical photon (Hagen, 1965; Salam, 1966; Taylor, 1967; Singleton, 1995).

Wilson and Guth argued that in a $U(1)$ gauge theory there should be a critical value of the coupling such that the theory undergoes a phase transition to a confining theory where the $U(1)$ gauge boson becomes massive. Combining this idea with the required large, nonperturbative magnetic charge which occurs in all monopole theories, and the electric–magnetic duality (which implies that it should not matter whether the nonperturbative coupling is electric or magnetic), we contend that the photon acquires a dynamical mass in the presence of magnetic charge. From an experimental point of view one can point to the $SU(3)$ theory of the strong interaction, which is thought to exist in the confining phase with a coupling constant that is considerably less than the coupling constant a magnetic monopole is required to have. The apparent experimental masslessness of the photon then implies the absence of Abelian magnetic monopoles of the Dirac or Wu–Yang type. A consistent monopole theory is still possible if one works with the Cabbibo–Ferrari theory *and* takes the somewhat unorthodox view that the second pseudo four-vector potential corresponds to a physical gauge boson.

The arguments given here should be taken strictly as applying only to Abelian monopoles. Objects like the 't Hooft–Polyakov monopole ('t Hooft, 1974; Polyakov, 1974), while also having an enormous magnetic charge, are of a somewhat different character than the Dirac or Wu–Yang monopoles. These magnetically charged objects come from an embedding of a $U(1)$ symmetry within a larger non-Abelian gauge group. Additionally, the magnetic charge of the theory is connected with the unusual topological structure of the Higgs field. Both of these facts make it difficult to formulate an electric–magnetic dual symmetry for the 't Hooft–Polyakov theory. Since this dual symmetry was crucial to our argument, we cannot use the arguments presented here to place any restrictions on the existence of 't Hooft–Polyakov magnetic charges.

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